Effective Field Theory & Decoupling in Multifield Inflation

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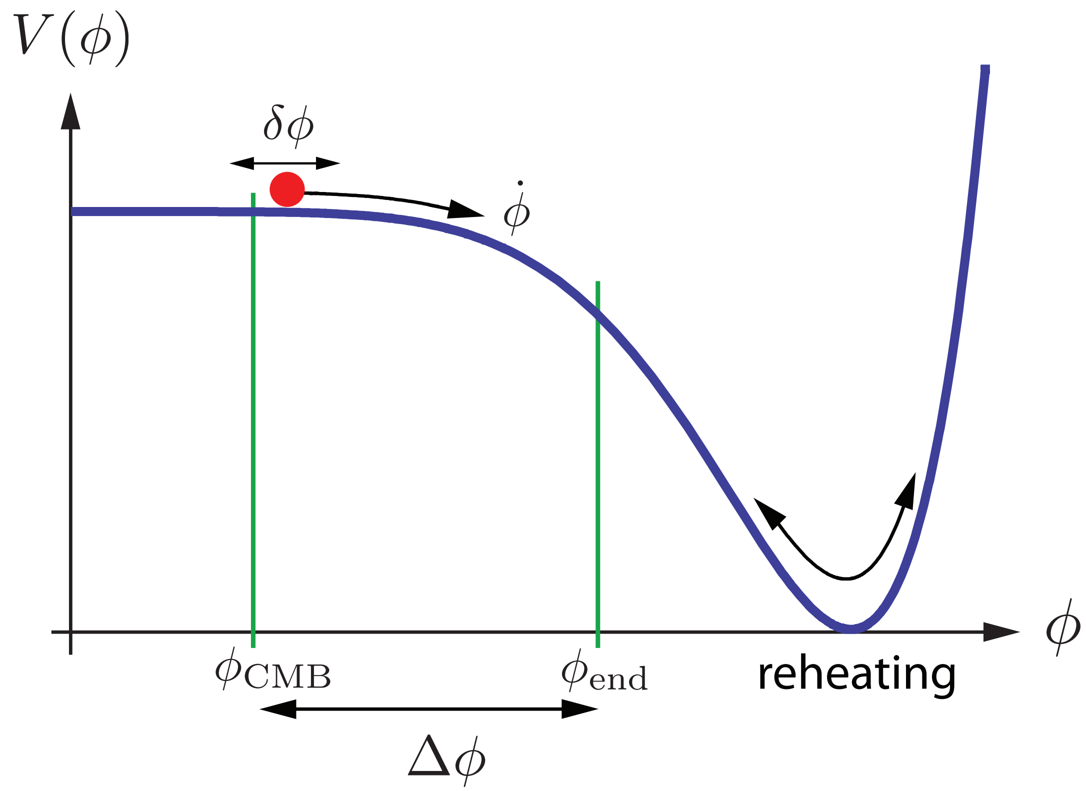
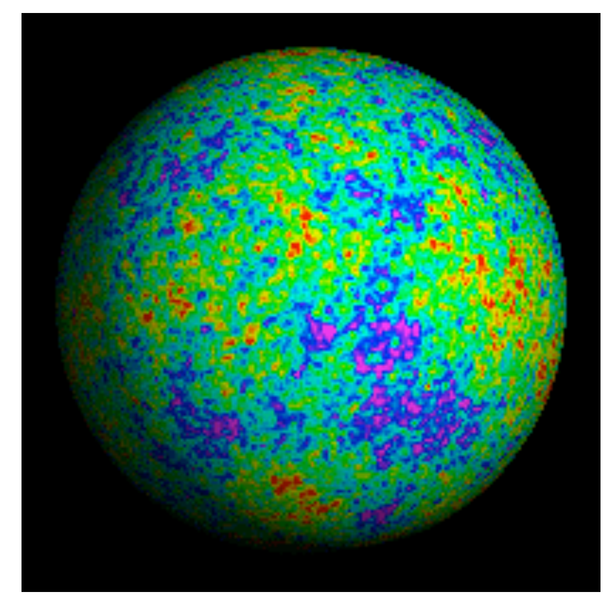
Work in progress with Jiajun Xu

Inflation as an EFT

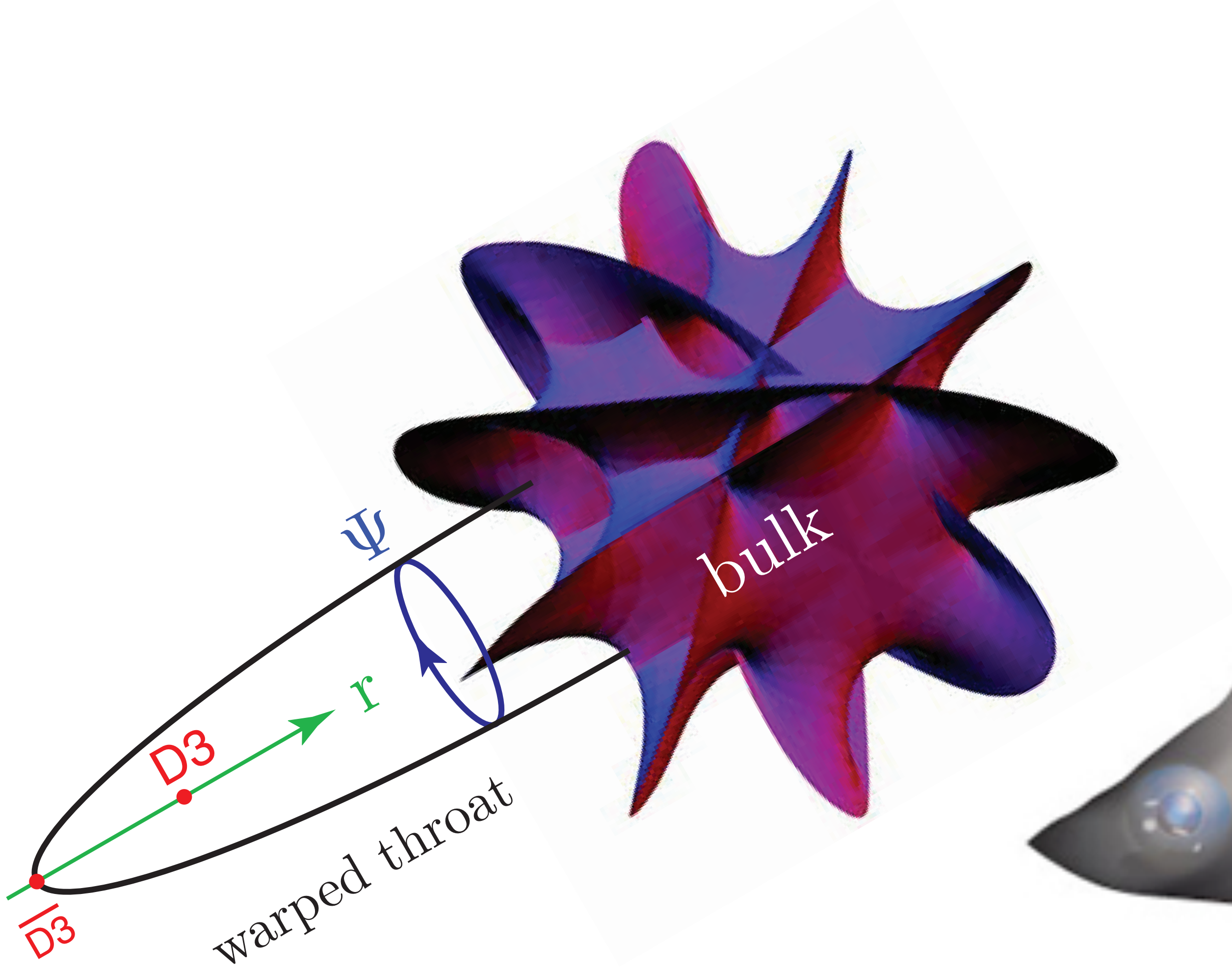
- Single “Order Parameter”: $\Phi(x,t)$

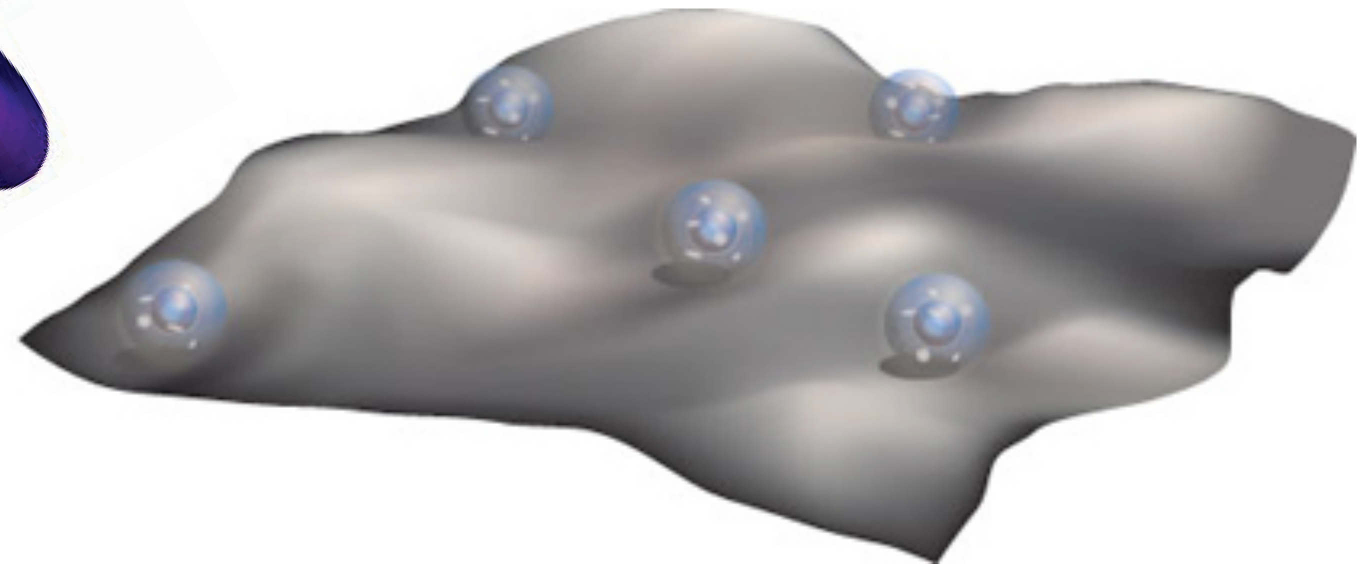




- UV incomplete: $\delta V \sim \frac{V}{M_P^2}\phi^2 \quad \Rightarrow \quad \eta \sim \mathcal{O}(1)$

Inflation in String Theory

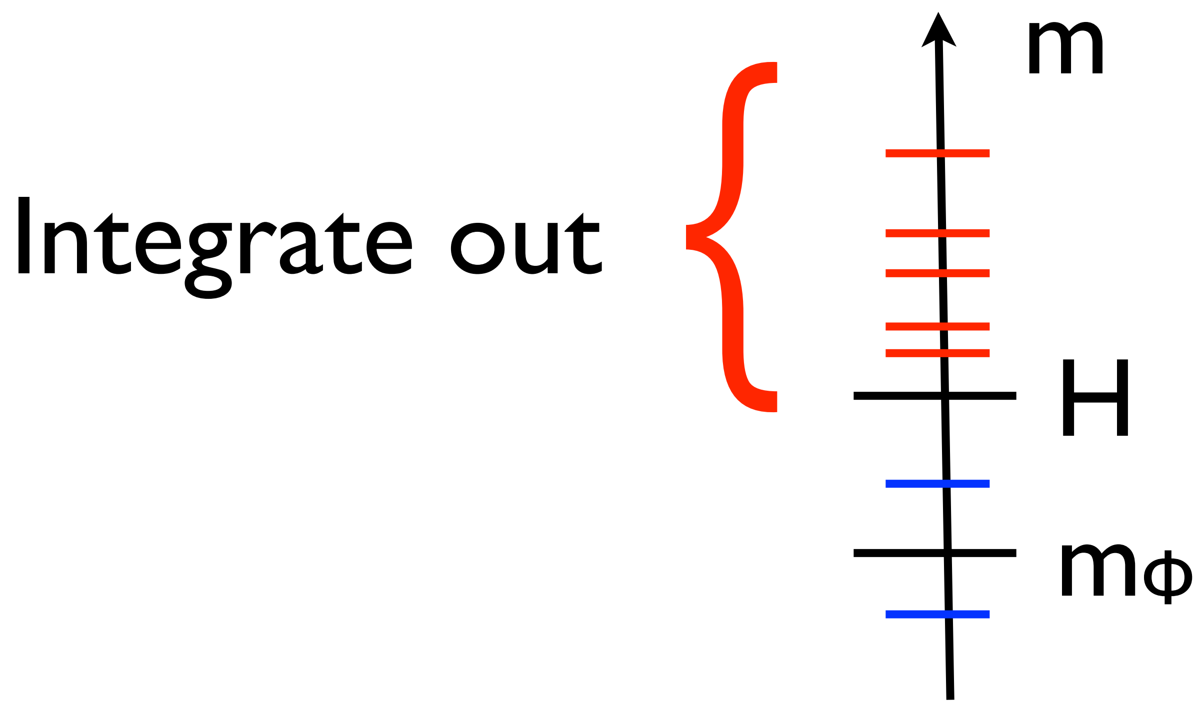




Rarely a single field model: many more field directions!

Conventional Wisdom

- If m> H, we can integrate them out:



- Only the light fields (m<H) contribute to curvature/ isocurvature perturbations.
- If only one with m< H, effective single field model.

Short Distance Scale

Slow-roll inflation:

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2} M_P^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] + \mathcal{O}\left(\frac{1}{M} \right)$$

In one Hubble time: $\Delta\phi = \dot{\phi} H^{-1} = \sqrt{2\epsilon} M_P$

For EFT truncation to finite powers of Φ/M:

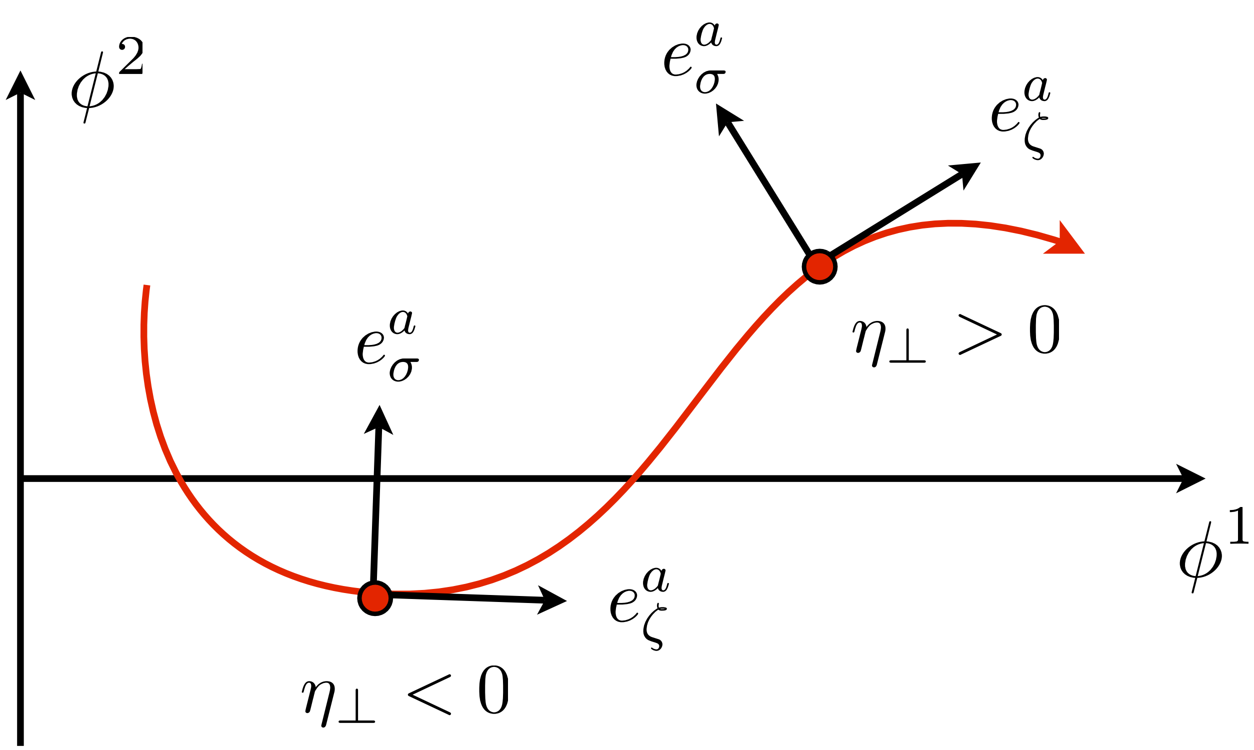
$$M >> \sqrt{2\epsilon} M_P$$

Weinberg, 08

Classical Background

Groot Nibbelink & van Tent

Multi-field: $\mathcal{D}_t\dot\phi^a \equiv \frac{\mathrm{d}\dot\phi^a}{\mathrm{d}t} + \Gamma^a_{bc}\,\dot\phi^b\dot\phi^c\ , \quad \Gamma^a_{bc} = \frac{1}{2}\gamma^{ad}\left(\gamma_{db,c}+\gamma_{dc,b}-\gamma_{bc,d}\right)$



Introduce vielbeins:

$$e^I_a e^J_b \delta_{IJ} = \gamma_{ab}\ , \quad e^I_a e^J_b \gamma^{ab} = \delta^{I.}$$

Choose: $e^a_\zeta \equiv \frac{\dot\phi^a}{\dot\phi_0}\ , \quad e^a_\sigma \equiv \frac{\mathcal{D}_t e^a_\zeta}{|\mathcal{D}_t e^a_\zeta|}$

& the rest denoted by

Composite field $\dot\phi_0^2 \equiv \gamma_{ab}\dot\phi^a\dot\phi^b$ **satisfies "single-field" EOM:**

$$\ddot\phi_0 + 3H\dot\phi_0 + \nabla_\parallel V = 0\ , \qquad \nabla_\parallel V \equiv \frac{\dot\phi^a}{\dot\phi_0}\nabla_a V$$

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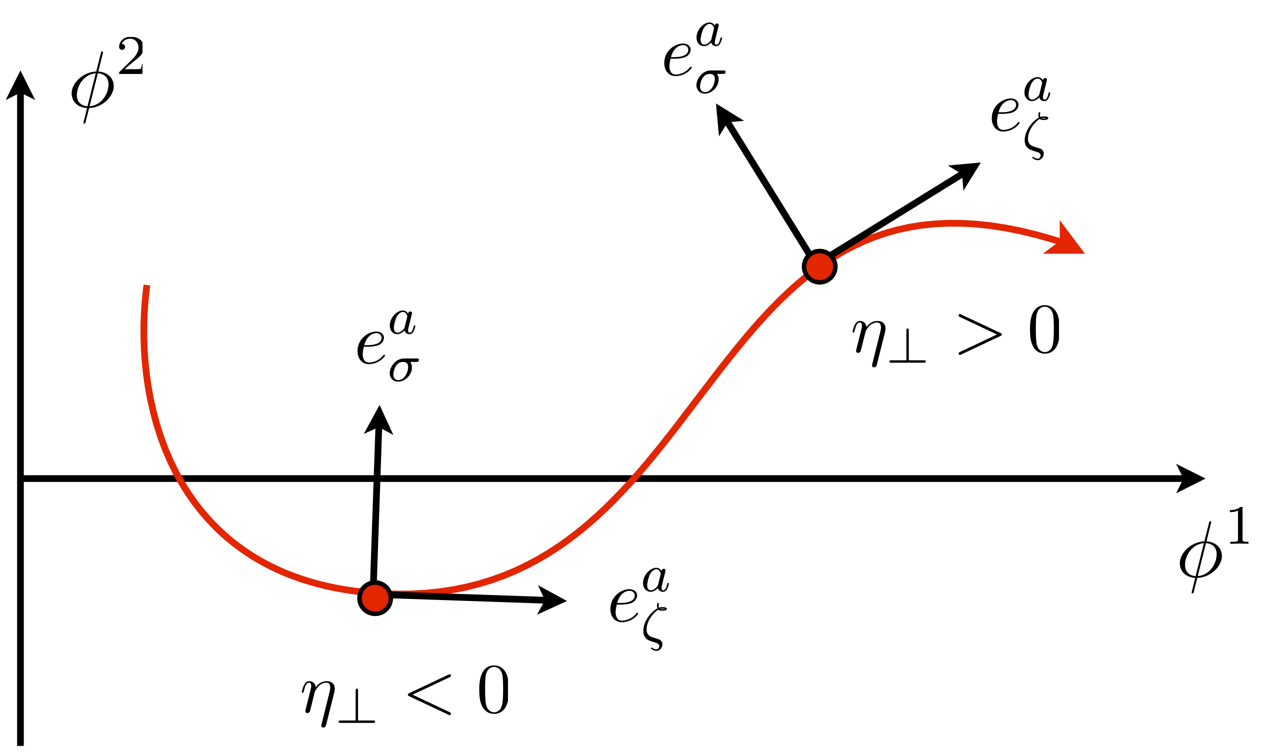

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& the rest denoted by e^a_s

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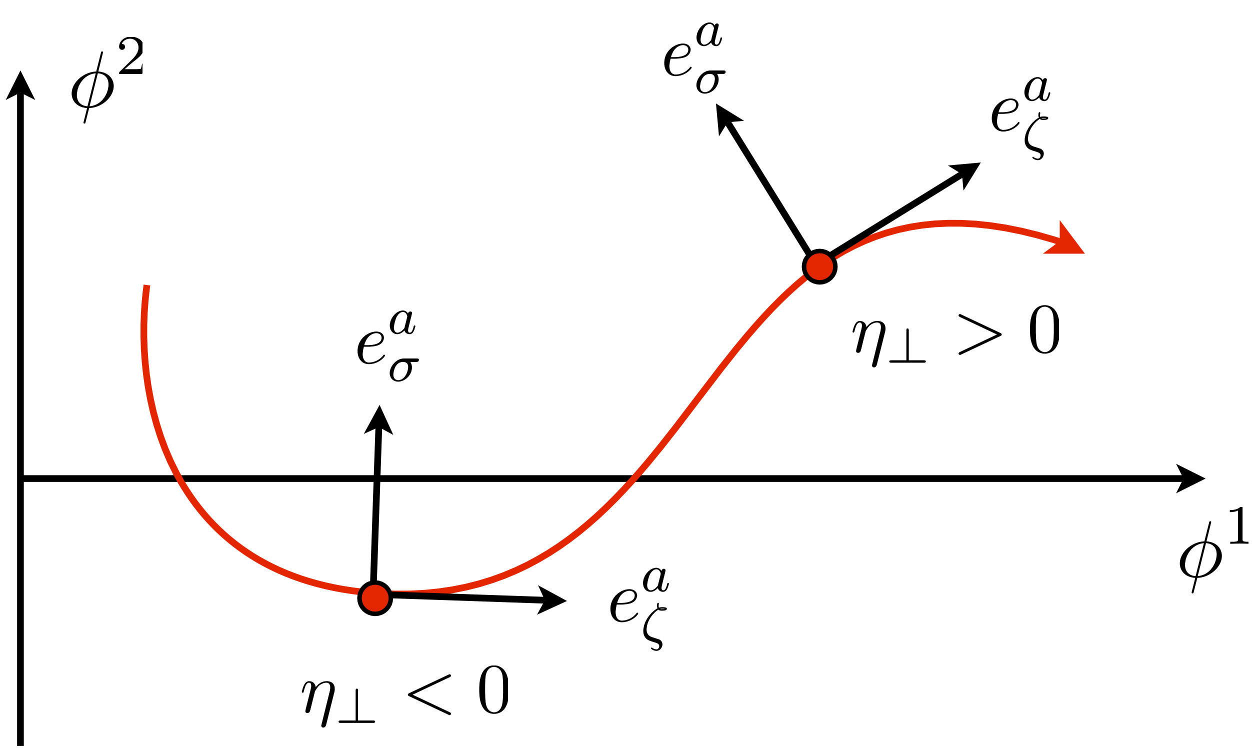

Turn rate:

$$\dot\theta = -\frac{e^a_\sigma\nabla_a V}{\dot\phi_0} = -\frac{\nabla_\sigma V}{\dot\phi_0}$$

centripetal force

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Mass Scales

- Mass scales tangent to classical trajectory:

$$M_{\|} \gtrsim \sqrt{2\epsilon} M_P$$

- Mass scales transverse to classical trajectory: no bound due to $\dot{\phi}_{\perp} = 0$ except for backreaction on ε:

$$\frac{M_{\perp}}{H} >> \frac{\dot{\theta}}{H}$$

rather weak! Heavy physics naively decoupled.

Quadratic Fluctuations

In terms of the veilbeins: $e^I_a e^J_b \delta_{IJ} = \gamma_{ab}\ , \quad e^I_a e^J_b \gamma^{ab} = \delta^{IJ}$

Define spin connection: $Y^I{}_J \equiv e^I_a D_t e^a_J\ .$

Quantum quadratic action can be expressed in terms of

$$Y^I_J, \dot{\theta},$$

and the mass matrix $m_{IJ} = e^a_I e^b_J m_{ab}$:

$$m_{ab} = M_{ab} - \frac{1}{a^3}\mathcal{D}_t\left[\frac{a^3\dot{\phi}_0^2}{H}e^\zeta_a e^\zeta_b\right]$$

$$M_{ab} \equiv \nabla_a\nabla_b V + 2\dot{H}\mathcal{R}_{acdb}\, e^c_\zeta e^d_\zeta\ .$$

Quadratic Fluctuations

In conformal time & properly normalizing the fluctuations:

$$\mathcal{L}^{(2)}_{(\zeta)} = \frac{1}{2}\left(v_\zeta'^2 - (\partial v_\zeta)^2 + \frac{z''}{z} v_\zeta^2\right) \tag{15}$$

$$\mathcal{L}^{(2)}_{(\sigma)} = \frac{1}{2}\left[v_\sigma'^2 - (\partial v_\sigma)^2 + \left(\frac{a''}{a} - \underline{a^2 M_{\sigma\sigma} + \theta'^2 - a^2 Y_\sigma{}^m Y_{m\sigma}}\right) v_\sigma^2\right] \tag{16}$$

$$\mathcal{L}^{(2)}_{(m)} = \frac{1}{2}\left[v_m'^2 - (\partial v_m)^2 + \left(\frac{a''}{a}\delta_{mn} - \underline{a^2 M_{mn} + a^2 Y^I{}_m Y_{In}}\right) v_m v_n + \underline{2a Y_{mn}(v_n v_m' - v_m v_n')}\right] \tag{17}$$

$$\mathcal{L}^{(2)}_{(\zeta,\sigma)} = \left(\underline{-2\theta' v_\sigma v_\zeta' + 2\frac{z'}{z}\theta' v_\sigma v_\zeta}\right) \tag{18}$$

$$\mathcal{L}^{(2)}_{(\sigma,m)} = \frac{1}{2}\underline{\left(-a^2 M_{\sigma m} + a^2 Y^I{}_\sigma Y_{Im}\right) v_\sigma v_m + a Y_{\sigma m}(v_m v_\sigma' - v_\sigma v_m')} \tag{19}$$

contains additional terms not present in the Goldstone approach of Senatore, Zaldarriaga. Imposing shift symmetries and high energy limit forbid many interesting contributions from turns in field space.

Two Field Model

General results simplified for models with two fields:

$$\mathcal{L}_0^{(2)} = \frac{1}{2}\left(v_\zeta'^2 - (\partial_i v_\zeta)^2 + \frac{z''}{z}v_\zeta^2\right) + \frac{1}{2}\left[v_\sigma'^2 - (\partial_i v_\sigma)^2 + \left(\frac{a''}{a} - a^2 M_{\sigma\sigma} + \theta'^2\right)v_\sigma^2\right]$$

$$\mathcal{L}_{\rm int}^{(2)} = -2\theta' v_\sigma v_\zeta' + 2\frac{z'}{z}\theta' v_\sigma v_\zeta$$

where $M_{\sigma\sigma} = V_{\sigma\sigma} + \epsilon H^2 \mathcal{R}$

Define: $$\eta_\| \equiv \frac{V_{\zeta\zeta}}{H^2}\ , \quad \eta_\perp \equiv \frac{M_{\sigma\sigma}}{H^2}\ , \quad \varrho \equiv \frac{\dot\theta}{H},$$

We can read off the "effective masses", c.f.,

$$\mathcal{L} = \frac{1}{2}\left(u'^2 - (\partial u)^2 + a^2 H^2\left(2 - \epsilon - \frac{m^2}{H^2}\right)u^2\right)$$

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$$\mathcal{L}_{\rm int}^{(2)} = -2\theta' v_\sigma v_\zeta' + 2\frac{z'}{z}\theta' v_\sigma v_\zeta$$

Effective masses:

$$m_\zeta^2 = H^2(\eta_\parallel - \varrho^2 - 6\epsilon - 2\epsilon\eta + 2\epsilon^2)$$

$$m_\sigma^2 = H^2(\eta_\perp - \varrho^2)$$

Several scenarios have been considered:

[Amendola, Gordon, Wands, Sasaki]; [Gordon, Wands, Bassett, Maartens]; [Peterson, Tegmark]; [Sasaki, Stewart]; [Venizzi, Wands]; [Meyers, Sivanandam]; [Garcia-Bellido, Wands];[Chen, Wang]; [Achucarro, Gong, Hardeman, Palma, Patil];[Cremonini, Lalak, Turzynski];[Baumann,Green]; ...

Two Field Model

I) Slow-roll Slow-turn (SRST): $\eta_{\|} \ll 1,\ \eta_{\perp} \ll 1 \text{ and } \varrho \ll 1.$

Two light fields, but can treat the interaction as perturbations

$\nabla\sigma$ sources superhorizon evolution of $\nabla\zeta$

Transfer functions: [Amendola, Gordon, Wands, Sasaki]; [Gordon, Wands, Bassett, Maartens]; [Peterson, Tegmark]

δN formalism: [Sasaki, Stewart]; [Venizzi, Wands]; [Meyers, Sivanandam]

The two approaches are equivalent [Garcia-Bellido, Wands]

Two Field Model

II) Quasi-single field: $\eta_{\|} \ll 1,\ \eta_{\perp} \sim 1$ and $\varrho \ll 1.$ [Chen, Wang]

A massive field which is critically damped, hence will decay (but slowly) after horizon exit.

Massive field can have large self-interactions which can mediate interaction among the light field through $\mathcal{L}^{(2)}_{\text{int}}$

Interaction part $\mathcal{L}^{(2)}_{\text{int}}$ can still be treated as perturbations.

Two Field Model

III) Effective Single-Field Limit $\eta_{\parallel} \ll 1,\ \eta_{\perp} \gg 1$

Conventional Wisdom: effectively a single field model

Turn in field space introduces interesting features:

$$c_s^{-2} \approx 1 + \frac{4\varrho^2}{\eta_{\perp} - \varrho^2 - 2 + k^2/(a^2 H^2)}$$ [Achucarro, Gong, Hardeman, Palma, Patil]

Sound speed is ill-defined when: $\varrho^2 > \eta_{\perp} >> 1$

In this limit, masses are comparable; also $\mathcal{L}_{\rm int}^{(2)}$ is significant, need to solve EOM of *full* quadratic action [Cremonini, Lalak, Turzynski]

Strong coupling scale for theories with a small sound speed.

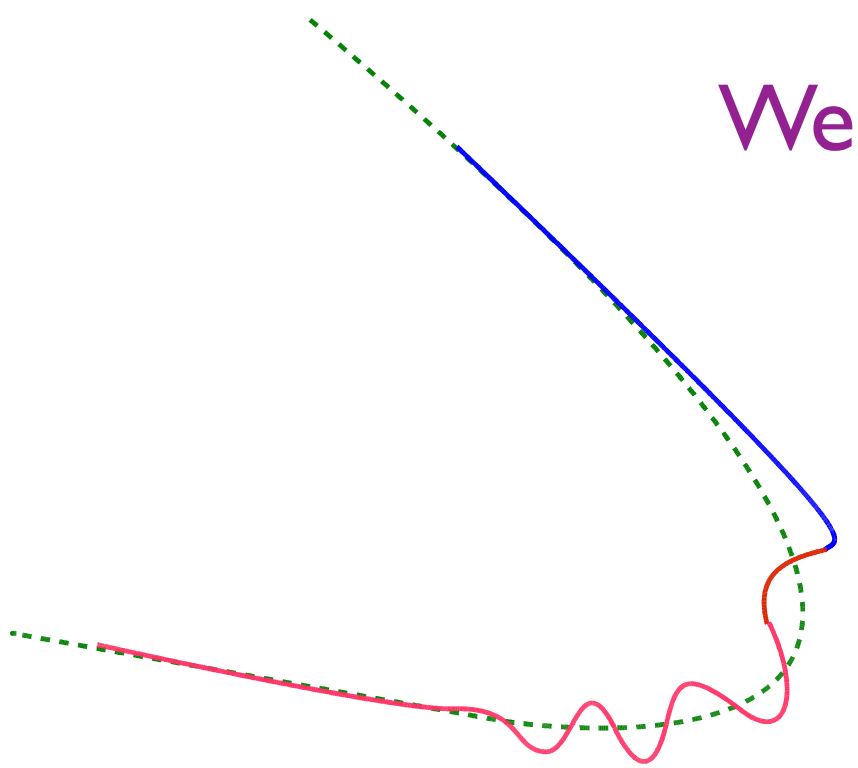
[Baumann, Green]

Sharp Turn in Two-Field Model

e.g., features in the potential or momentarily large kinetic mixing

For the backreaction of the turn to be small: $\eta_\perp > \rho^2$

Momentarily large ρ leads to (i) sudden change in masses, (ii) projection of perturbations along σ to the inflation direction.



We focus on the effects of sharp turn

Subsequent oscillations of the massive field recently studied by Chen (trigger resonant non-Gaussianity).

Sharp Turn in Two-Field Model

The EOMs for the coupled system are:

$$\frac{\mathrm{d}^2 v_\zeta}{\mathrm{d}x^2} + \left(1 - \frac{2}{x^2}\right) v_\zeta - \frac{2\varrho}{x^2} v_\sigma + \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{2\varrho}{x} v_\sigma\right) = 0$$

$$\frac{\mathrm{d}^2 v_\sigma}{\mathrm{d}x^2} + \left(1 - \frac{2 - \eta_\perp + \varrho^2}{x^2}\right) v_\sigma - \frac{4\varrho}{x^2} v_\zeta - \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{2\varrho}{x} v_\zeta\right) = 0$$

Momentary turn: $\varrho = \frac{\dot{\theta}}{H} = \frac{\Delta\theta}{H}\,\delta(t - t_0) = \Delta\theta\, x_0\, \delta(x - x_0)$ $\qquad x \equiv k\tau$

Matching b.c.:

$$v_\zeta(x < x_0) = v^+(k,\tau)\ ,$$

$$v_\zeta(x > x_0) = C_1 v^+(k,\tau) + C_2 v^-(k,\tau)\ ,$$

$$v^\pm(k,\tau) = \frac{-1}{\sqrt{2k}} e^{\mp i x}\left(\frac{1}{x} \pm i\right)\ .$$

gives:

$$C_1 = 1 - \frac{\Delta\theta}{x_0} e^{i x_0}\left(1 + \frac{i}{x_0}\right)\sqrt{2k}\, v_\sigma(x_0)\ ,$$

$$C_2 = -\frac{\Delta\theta}{x_0} e^{-i x_0}\left(1 - \frac{i}{x_0}\right)\sqrt{2k}\, v_\sigma(x_0)\ .$$

Power Spectrum

Similar to initial state effect on inflationary spectrum:

$$P_\zeta = \frac{k^3}{2\pi^2} \left| \frac{v_\zeta}{a\sqrt{2\epsilon}} \right|^2_{x\to 0} = \frac{H^2}{8\pi^2\epsilon} |C_1 + C_2|^2$$

not only a change in sound speed!

In the small x_0 limit: $|C_1 + C_2| \to 1 \ , \quad x_0 \to 0 \ .$

In the large x_0 limit: $|C_1 + C_2|^2 \approx 1 + 2\Delta\theta \frac{\sin(2x_0)}{x_0} \ , \quad x_0 \gg \Delta\theta, \, \eta_\perp$

even the sharp turn happens more than 60e-folds before inflation ends, it may still leave an imprint.

3-point functions

- Following the standard method: $\langle\zeta^3\rangle = -i\int dt\langle[\zeta^3, H_I(t)]\rangle$

$$\zeta(\mathbf{k},\tau) \equiv \frac{v_\zeta(\mathbf{k},\tau)}{a\sqrt{2\epsilon}} = u(\mathbf{k},\tau)a_{\mathbf{k}} + u^*(-\mathbf{k},\tau)a^\dagger_{-\mathbf{k}}$$

- Consider a simple interaction vertex

$$H_I = -\int dx^3 a^3 \epsilon^2 \zeta\zeta'^2$$

$$\langle\zeta^3\rangle = i(u_{\mathbf{k}_1}u_{\mathbf{k}_2}u_{\mathbf{k}_3})|_{\tau=0}\int_{-\infty}^{0} d\tau\, a^2\epsilon^2\, u^*_{\mathbf{k}_1}(\tau)\frac{u^*_{\mathbf{k}_2}(\tau)}{d\tau}\frac{u^*_{\mathbf{k}_3}(\tau)}{d\tau}\ldots$$

- Non Bunch-Davis correction: flip the sign of one of the momentum, with overall factor $|C_2|^2$

$f_{NL} \sim \mathcal{O}(\epsilon)|C_2|^2$ peaked in the folded limit $k_1 + k_2 = k_3$

3-point functions

- Generically the signal in 3-pt function is small oscillation in the power spectrum ~ $|C_2| \lesssim 0.1$

$$f_{NL} \sim \mathcal{O}(\epsilon)|C_2|^2 \sim 10^{-4}$$

- After the turn, the massive field is generically oscillating $\Rightarrow$ resonant enhancement of 3-point function

[Chen, Easther, Lim (2008)] [Chen 2011]

$$H_I = -\int \mathrm{d}\tau \mathrm{d}x^3 \frac{1}{2} a^2 \epsilon \dot{\eta}\, \zeta^2 \zeta'$$

$$f_{\rm NL}^{\rm res}|_{\text{non BD}} \sim \frac{\sqrt{\pi}}{8} \beta \left(\frac{M_\sigma}{H}\right)^{5/2} \left(\Delta\theta \frac{k_0}{k_1}\right)^2 \sin\left(\frac{2M_\sigma}{H} \ln \tilde{K}_1 + \phi\right) + \text{perm} \qquad \tilde{K}_i = K - 2k_i$$

$$\beta \ll 1\,, \quad M_\sigma \gg H\,, \quad \beta \frac{M_\sigma^2}{H^2} \gg 1$$

3-point functions

- Comparing with result based on Bunch-Davis state

$$f_{NL}^{\rm res}|_{\rm BD} \sim \sin\left(\frac{\omega}{H}\ln K + {\rm phase}\right)$$
$$f_{NL}^{\rm res}|_{\rm non\ BD} \sim \sin\left(\frac{\omega}{H}\ln \tilde{K}_i + {\rm phase}\right)$$

- A new origin of non BD component - Sharp turn

c.f. [Chen 2010]

- Role of the massive field:
  - ❖ Provides non BD component through sharp turn
  - ❖ Provides oscillating time dependent background to trigger resonant effect

3-point functions

- If the inflaton action is p(X) $X \equiv \gamma_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$

$$H_I = -\int \mathrm{d}\tau \mathrm{d}x^3 \frac{a\epsilon}{Hc_s^2}\left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma}\right)\zeta'^3$$

$$f^{\mathrm{res}}_{\mathrm{N}L\ 2}\big|_{\mathrm{non\ BD}} \sim \left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma}\right)\left(\frac{c_s^2}{\epsilon}\right)_0 \left(\frac{\omega}{H}\right)^{5/2}\left(\frac{\Delta\theta}{x_{0i}}\right)^2$$

Effects further enhanced by the small sound speed!

Summary

- Effective field theory and decoupling of massive modes becomes more subtle in multi-field models.
- Strong bound on M for single field models from classical dynamics (c.f. Weinberg) can be relaxed.
- Quantum fluctuations of *massive* fields may leave imprints on light field (curvature mode), both in the power spectrum and in non-Gaussianity.
- Such effects are not captured by the Goldstone mode method of Senatore and Zaldarriaga without breaking the shift symmetry.